RESEARCH DEPARTMENT

ABSOLUTE MEASUREMENTS IN MAGNETIC RECORDING

Report No. C-089

(1956/4)

W. Proctor Wilson

E.D. Daniel, M.A., A.M.I.E.E.
P.E. Axon, O.B.E., M.Sc., Ph.D., A.M.I.E.E.

(W. Proctor Wilson)

ABSOLUTE MEASUREMENTS IN MAGNETIC RECORDING

List of Symbols Used in the Analysis

A = Cross-sectional area of head gap

A_c = Cross-sectional area of head core

b = Gaplength of head or width of non-magnetic conductor

B_y = Surface induction (normal component of induction at tape surface in free space)

c = Thickness of tape magnetic coating

d = Depth of the non-magnetic conductor

E = Reproduced e.m.f.

H_x = Intensity of recording field

I = Signal current in recording head

J_x = Intensity of magnetisation in the tape

K = Head sensitivity (H_x/I)

N = Number of turns on head

r = Total reluctance of head core and gaps

v = Tape speed

w = Tape width

η = Tape sensitivity (J_x/H_x)

λ = Wavelength of recorded signal

μ = Tape permeability

μ_c = Permeability of the head core

ϕ = Flux

Electromagnetic units are used throughout unless otherwise stated.

February 1956

Report No. C-089

(1956/4)

ABSOLUTE MEASUREMENTS IN MAGNETIC RECORDING

SUMMARY

It is shown that an absolute measure of the surface induction of a recorded tape can be made using a non-magnetic conductor of the type developed in Research Department in connection with frequency-characteristic standardisation. Following on this, an analysis of the long-wavelength response of a magnetic recording system is given which leads to a method of defining the absolute sensitivity of a tape in terms of the surface induction created under certain measurable conditions. The results obtained may prove of value in developing satisfactory techniques for recording level standardisation and tape specification.

1. INTRODUCTION.

The most difficult aspect of standardisation in magnetic recording, namely frequency-characteristic standardisation, has received considerable attention and procedures have now been established[1] whereby a machine can be adjusted to conform to an agreed frequency characteristic within satisfactory tolerances. Certain basic definitions have been adopted in formulating these proposals. Thus the fundamental overall response or sensitivity of a tape recorder is defined as the ratio E/I, where I, the signal current in the recording head, and E, the reproduced e.m.f. corresponding to I, are both easily determined quantities. It is logical to express the overall sensitivity as the produce of a recording and a reproducing sensitivity but this necessitates introducing a third quantity indicative of the magnetic state of the recorded tape. The quantity which has been accepted for this purpose is the magnetic induction B_y normal to the surface of the recorded tape when out of contact with the magnetic heads. Thus the recording sensitivity is defined as the ratio B_y/I and the reproducing sensitivity is defined as the ratio E/B_y. In connection with frequency characteristic standardisation, the frequency-variation of these quantities, rather than their absolute values, has been of interest.

This report deals first with the question of measuring the absolute magnitude of surface induction—a necessary adjunct to the standardisation of recorded level. Secondly, consideration is given to the possibility of measuring the absolute sensitivity of the various parts of the recording-reproducing chain, with particular reference to formulating a reasonable unequivocal representation of "tape sensitivity". Hitherto there has been no established method of quoting tape sensitivity other than by comparing one tape with another, well known, variety.

The measurement of surface induction B_y at medium wavelengths presents no great difficulty. A direct measurement can be made using a non-magnetic conductor of exactly the same type as that previously described in connection with frequency characteristic standardisation[2].

The question of determining tape sensitivity is more complex since, at first sight, it must involve measurement of the recording field. It can be shown, however, that a direct measurement of recording field is unnecessary if the system obeys the form of reciprocity suggested by Westmijze[3]. An application of this principle enables the sensitivities of both a head and a tape to be expressed in terms of a measured value of surface induction, and the recording current and reproduced e.m.f. associated with this value of induction. This assumes that the same head, of specified gaplength, is used for both recording and reproducing. The particular definitions of head and tape sensitivities suggested are, it is believed, those most clearly indicative of the physical processes involved.

2. MEASUREMENT OF SURFACE INDUCTION.

The only completely satisfactory known method of measuring surface induction is by means of a non-magnetic conductor head. An analysis of the action and details of the construction of this device have been given elsewhere[2].

When a sinusoidally recorded tape is moved across the correctly aligned conductor, an e.m.f. is produced between the ends of the conductor which is related to the value of surface induction by the expression

$$E = vwB_y \cdot \frac{1 - \exp(-2\pi d/\lambda)}{\lambda/2\pi d} \cdot \frac{\sin(\pi b/\lambda)}{\pi b/\lambda} \qquad (1)$$

the position of the various dimensions being illustrated in Fig. 1.

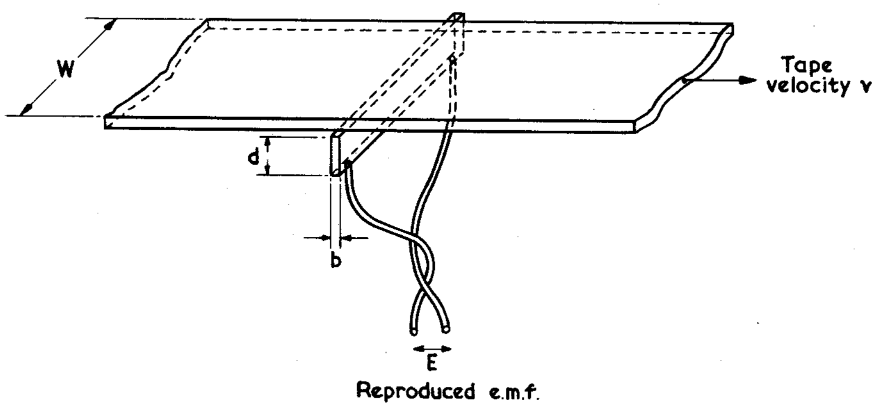

Fig. 1 - Reproduction by means of a conductor head

Experiments using conductors of varying cross-section in contact with the tape have been shown[2] to provide the same measure of surface induction providing that the correct relative constants are embodied in the expression defining the head output. In the present instance an absolute measurement of e.m.f. is required at only one wavelength which can be chosen so that the conditions $d > \lambda$ and $\lambda \gg b$ are easily fulfilled. The value of induction is then given closely by

$$B_y = 2\pi d E/vw\lambda \qquad (2)$$

A reliable calculation of induction can be made using this expression provided the depth of the conductor is precisely known and care is taken, when determining the e.m.f., to carry out an accurate calibration of the transformer and high-gain reproducing amplifier. If the conductor head and transformer are suitably designed the signal/noise ratio should be quite adequate.

To illustrate the method and to obtain an idea of the magnitude of surface induction which may be encountered, a measurement was made on a tape recorded at 15 in./sec with a 1 kc/s signal at a standard level of an order sometimes used in practice.

The dimensions of the conductor were: d = 37·9 mil (0·096 cm), b = 0·5 mil (0·00127 cm). The wavelength of a 1 kc/s signal recorded at 15 in./sec is 15 mil (0·0381 cm), so that the use of the approximate equation (2) is justified.

The r.m.s. value of the e.m.f. between the ends of the conductor was found to be 0·0435μV(4·35 e.m.u.). This corresponds to an r.m.s. value of surface induction given by

$$B_y = 2{\cdot}84 \text{ gauss}$$

This value corresponds to a recorded level more than 20 dB below the overload point of modern tapes. Recent American measurements[4] giving a value of some 20 gauss obviously correspond to a level much nearer this point.

It is of interest to work out the algebraic sum of the flux emanating from a wavelength on the tape recorded at the lower level. Using a loss-free recording system the flux would be the same at all wavelengths, since surface induction is inversely proportional to wavelength. The flux, Φ_λ, from one wavelength will be given by

$$\Phi_\lambda = 2w \int_0^{\lambda/2} B_y \cos(2\pi x/\lambda)\, dx$$

Using the above figures this gives a value

$$\Phi_\lambda = 0{\cdot}031 \text{ line}$$

which represents the maximum amount of flux fed to a head during reproduction of a tape recorded with the lower standard level. Thus during the reproduction of low level signals, which may be some 50 dB below the standard level, the head can receive a flux of less than a hundredth of a line and still provide an acceptable signal-to-noise ratio.

3. MEASUREMENT OF HEAD AND TAPE SENSITIVITIES.

3.1. Basic Analysis and Definitions.

The analysis of the recording-reproducing process here given is valid only for reasonably long wavelengths, i.e. for wavelengths considerably greater than

gaplength or tape thickness but small compared with the overall dimensions of the head and the width of the tape. Thus it will be assumed that in traversing the recording-head gap, an element of tape experiences a longitudinal signal field which is of uniform intensity within the precincts of the gap and zero outside these limits. The actual mechanisms of h.f. biasing will be ignored and the bias regarded purely as a "linearising catalyst" in the recording process.

In the absence of the tape, let the recording field a distance y above the head surface, be given by

$$H_x = KI \tag{3}$$

where K, the "head sensitivity" is a function of the core shape and permeability, the number of turns in the magnetising coil and the gaplength. The effect of bringing the tape into contact with the head is to reduce H_x by a factor which is approximately given by $A/(A+\mu wc)$. However, since $wc \ll A$ and μ is not likely to be greater than 5, the reduction in H_x is quite negligible and it is safe to assume that (3) also gives the recording field in the presence of the tape.

Thus, referring to Fig. 2, the recording flux created in an element of tape of area $w\delta y$ by a recording current I is given by

$$\delta\phi = \mu H_x \,.\, w\delta y = \mu KI \,.\, w\delta y \tag{4}$$

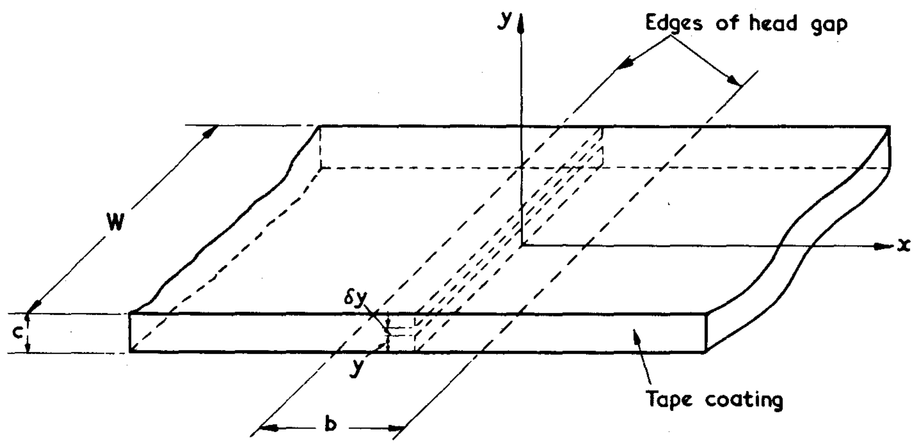

Fig. 2 - Geometry of system used in basic analysis

When the element leaves the gap this flux will fall to some remanent value corresponding to a remanent intensity of magnetisation J_x. Since, when bias is used, the recording process is known to be accurately linear, J_x must be proportional to H_x. Let the "tape sensitivity", η, be defined by the equation

$$J_x = \eta H_x = \eta KI \tag{5}$$

Then, on leaving the gap, an element of tape of area $w\delta y$ and length δx has a magnetic moment given by

$$\delta M = \frac{J_x}{\mu} \cdot w\,\delta x\,\delta y$$

$$= \frac{\eta K I}{\mu} \cdot w\,\delta x\,\delta y \qquad (6)$$

The problem now is to calculate what flux this element excites in the core of the same head subsequently used as a reproducing head. In order to do this, a reciprocity principle will be used which can be conveniently stated as follows:

If a current I through a head coil of N turns excites in an element of tape of given area a flux $\delta\phi$, then a current NI flowing round the element will excite a flux of the same value $\delta\phi$ in the head core.

In the element of tape considered above, the magnetisation is equivalent to a peripheral current of magnitude

$$I' = \frac{\delta M}{w\,\delta y} = \frac{\eta K I}{\mu} \cdot \delta x \qquad (7)$$

On reproduction, therefore, the element excites in the head core a flux

$$\delta\phi' = \frac{I'}{NI} \cdot \delta\phi = \frac{\eta K^2 I w}{N} \cdot \delta x\,\delta y \qquad (8)$$

Hence the flux produced by the whole tape is given by

$$\phi' = \frac{w}{N} \int_{-\infty}^{+\infty} \mathrm{d}x \int_0^c \eta K^2 I\,\mathrm{d}y \qquad (9)$$

If λ is sufficiently greater than b for I to remain substantially constant while an element of tape traverses the gap, then equation (9) can be written

$$\phi = \frac{wI}{N} \int_{-\infty}^{+\infty} \mathrm{d}x \int_0^c \eta K^2\,\mathrm{d}y$$

Finally it is to be assumed that K is uniform within the gap and zero outside and is constant throughout the tape thickness. The latter assumption implies that η is also constant throughout the thickness and the flux is, therefore, given by

$$\phi' = \frac{wI}{N} \cdot \eta K^2 \int_{-b/2}^{+b/2} \mathrm{d}x \int_0^c \mathrm{d}y \qquad (10)$$

whence

$$\phi' = \frac{wcb\eta K^2 I}{N} \qquad (11)$$

The validity of this last assumption is discussed further at a later stage. The reproduced e.m.f. is given by

$$E = -\frac{N\,\mathrm{d}\phi'}{\mathrm{d}t} = -vN\frac{\mathrm{d}\phi'}{\mathrm{d}x}$$

or, regarding E and ϕ as the r.m.s. values of e.m.f. and flux,

$$E = \frac{2\pi v N \phi}{\lambda}$$

so that, substituting for ϕ from (11)

$$E = \frac{2\pi\, vwcb\eta K^2 I}{\lambda} \tag{12}$$

or the "overall sensitivity" of the system is given by

$$\frac{E}{I} = \frac{2\pi\, vwcb\eta K^2}{\lambda} \tag{13}$$

As might be expected, this shows that the overall sensitivity of the system is proportional to the tape speed, the tape sensitivity, the square of the head sensitivity and the cross-sectional area of the tape, but inversely proportional to the wavelength of the recorded signal.

The next step in the analysis is to separate the overall sensitivity into the sensitivities of the recording and reproducing processes. This means introducing the concept of "surface induction", B_y, which is now generally accepted as the quantity to be used to define the magnitude of the recorded signal. Now, if $\lambda \gg c$, the relation between B_y and J_x is given to a close approximation by[5]

$$B_y = \frac{4\pi^2 c}{\lambda} \cdot J_x \tag{14}$$

for a sinusoidally recorded tape in free space. This relation was deduced assuming a tape permeability of unity but it can be shown[3] that the values of permeability likely to be encountered in practice should have a quite negligible effect at long wavelengths. Putting $\eta K I = J_x$ in (12) gives

$$E = \frac{2\pi vwcbK}{\lambda} \cdot J_x \tag{15}$$

so that substituting for J_x from (14), the "reproducing sensitivity" of the system is given by

$$\frac{E}{B_y} = \frac{vwbK}{2\pi} \tag{16}$$

Eliminating E from (13) and (16) gives the "recording sensitivity" as

$$\frac{B_y}{I} = \frac{4\pi^2 c\eta K}{\lambda} \tag{17}$$

The reproducing sensitivity is thus shown to be proportional to tape speed, head sensitivity, tape width, and gaplength. Since in most heads the major proportion of the reluctance is in the gap, K is approximately inversely proportional to b,

and the reproducing head sensitivity tends to be independent of gaplength. It is, naturally enough, independent of tape thickness——one of the advantages of expressing the strength of a recorded signal in terms of surface induction rather than, say, intensity of magnetisation which has been, at times, suggested. The recording sensitivity, on the other hand, is proportional to tape sensitivity, head sensitivity and tape thickness. It is, of course, inversely proportional to wavelength which is another way of stating that constant recording current creates in an ideal system an induction rising at 6 dB/octave with signal frequency.

Finally it is clear from (16) and (17) that the sensitivities of the head and tape can be expressed in terms of measurable quantities. Thus the head sensitivity is given by

$$K = \frac{2\pi}{vwb} \cdot \frac{\mathbf{E}}{\mathbf{B}_y} \qquad (18)$$

and the tape sensitivity by

$$\eta = \frac{v\lambda w\, b}{8\pi^2 c} \cdot \frac{\mathbf{B}_y^2}{\mathbf{E}I} \qquad (19)$$

An alternative derivation of these expressions is given in the Appendix.

3.2. Specification of Gaplength of the Head.

The weak point in the foregoing formulation of tape sensitivity is that it is based upon the assumption that the strength of the recording field does not decrease through the thickness of the tape. For a given gaplength the way in which the field intensity in the central region of the gap decreases through the tape thickness can be calculated[3] and an attempt could be made to carry out the integration of (8) taking this into account. Such an attempt would, however, be meaningless unless the corresponding fall-off in bias field and its effect on tape sensitivity were included and this, as has been shown elsewhere[5] leads to results of such complexity that an acceptable formulation of tape sensitivity would no longer be practicable.

The alternative is to specify the gaplength of the head so that, although non-uniform, the distribution of recording field is always the same through a given tape thickness. The symbols η and J may then be regarded as representing the mean values of tape sensitivity and intensity of magnetisation through the tape thickness. The suggested value of gaplength for this purpose is 1 mil (0·00254 cm). This, while small compared with the standard wavelength of 15 mil (0·0381 cm), corresponding to a 1 kc/s tone recorded at 15 in./sec, is sufficiently large to ensure that the recording field distribution does not fall too rapidly through the thickness of the average coated tape (the range likely to be encountered is of the order of 0·4 to 0·6 mil). It will be well, however, if a practice is made of always giving the coating thickness when quoting tape sensitivities.

As a general check on the validity of the procedure, measurements were made of the relative recording and reproducing sensitivity of three ring-type reproducing-recording heads having a standard gaplength of 1 mil but otherwise with important differences. Head A was a conventional commercial reproducing head with tapered

pole-tips; Head B was a commercial recording head used back-to-front so that the pole-tips were not tapered; Head C was a conventional commercial recording head. If the general conclusions resulting from the reciprocity principle are correct, the relative sensitivities of the heads should be the same when used for recording or reproducing. The Table below, giving the results obtained, shows this to have been the case, to within the limits of accuracy to be expected.

Head	Relative Recording Sensitivity	Relative Reproducing Sensitivity
	dB	dB
A	0	0
B	−15·2	−14·4
C	−5·5	−5·0

3.3. Suggested Procedure.

The suggested practical procedure is as follows:

i. A conventional type recording head is dismantled and reassembled with gap spacer of 1 mil (0·00254 cm).

ii. A recording is made, using this head and the tape under test, of a 1 kc/s signal at 15 in./sec (38·1 cm/sec), with the bias adjusted to give maximum sensitivity. The recording level is not critical so long as non-linearity is avoided. The recording current, I, used is measured.

iii. The r.m.s. value of the surface induction, B_y, recorded on the tape is measured by means of a non-magnetic conductor head, as previously described.

iv. The tape is again reproduced using the recording head now as a reproducing head and the open circuit voltage, E, across it is measured.

v. With the values of I, B_y and E so measured the tape sensitivity can be calculated from the relation

$$\eta = 740 \cdot \frac{B_y^2}{EI} \text{ e.m.u.} \qquad (20)$$

where B_y is in gauss, I in microamperes and E in microvolts. The assumed constants of the system are:

$$\begin{aligned} v &= 15 \text{ in./sec } (38{\cdot}1 \text{ cm/sec}) \\ \lambda &= 15 \text{ mil } (0{\cdot}0381 \text{ cm}) \\ w &= 0{\cdot}25 \text{ in. } (0{\cdot}0636 \text{ cm}) \\ b &= 1 \text{ mil } (0{\cdot}00254 \text{ cm}) \\ \text{and } c &= 0{\cdot}5 \text{ mil } (0{\cdot}00127 \text{ cm}) \end{aligned}$$

If the thickness of the tape coating differs appreciably from the value of 0·5 mil the appropriate correction should be made.

4. REFERENCES.

1. Documents of the VIIth Plenary Assembly, C.C.I.R., London, 1953.

2. Daniel, E.D. and Axon, P.E., "The Reproduction of Signals Recorded on Magnetic Tape", 1953, Proceedings of the I.E.E., Vol. 100, Part III, No. 65, May 1953, pp. 157–167.

3. Westmijze, W.K., "Studies on Magnetic Recording", 1953, Philips Research Reports, Vol. 8, pp. 161–183, 245–269.

4. Schwartz, R., Sheldon, I.E. and Comerci, F.H., "Absolute Measurement of Signal Strength on Magnetic Recording", J.S.M.P.T.E., Vol. 64, No. 1, January 1955, pp. 1–5.

5. Daniel, E.D., "The Influence of Some Head and Tape Constants on the Signal Recorded on Magnetic Tape", Proceedings of the I.E.E., Vol. 100, Part III, No. 65, May 1953, pp. 168–175.

APPENDIX

It is of interest to observe that an alternative, though not so rigorous derivation of (18) and (19) is possible, in which the reciprocity principle is not explicitly stated. Instead, use is made of an expression for the reproducing head response which derives from a consideration of the reluctance of the paths open to the tape flux upon entering the core. The full derivation of this expression is given elsewhere[2]. The result can be written in the form:

$$\frac{E}{B_y} = 2\pi N w \cdot \frac{1/A - 1/\mu_c A_c}{r} \cdot \frac{\lambda}{\pi} \sin \frac{\pi b}{\lambda} \tag{21}$$

where the factors $1/A$ and $1/\mu_c A_c$ are, in effect, the reluctances per unit length of the gap and core respectively, and r is the total reluctance of the core including that of a possible rear gap. In all practical cases $A_c \gg A$ and $\mu_c \gg 1$ so that $1/A \gg 1/\mu_c A_c$. Also, in the present instance only long wavelengths are of interest, so that $(\lambda/\pi) \sin (\pi b/\lambda) = b$. To a close approximation, therefore, (21) can be written

$$\frac{E}{B_y} = \frac{2Nvwb}{Ar} \tag{22}$$

Now when the same head is used as a recording head, a current I through the coil will produce a field in the gap of value

$$H_o = \frac{4\pi NI}{Ar} \tag{23}$$

Let the field strength H_x, a distance y above the head surface, be given by

$$H_x = aH_o \tag{24}$$

where "a" can be regarded as a leakage factor of value less than unity. Then from (23) and (24) the head sensitivity is given by

$$K = \frac{H_x}{I} \qquad \frac{4\pi aN}{Ar}$$

or combining this with (22)

$$K = \frac{2\pi a}{vwb} \cdot \frac{E}{B_y} \tag{25}$$

Now by definition, the tape sensitivity is given by

$$\eta = \frac{J_x}{H_x}$$

Putting $H_x = KI$ and making use of (14) to write $J_x = \lambda B_y/4\pi^2 c$, this becomes

$$\eta = \frac{\lambda}{4\pi^2 cK} \cdot \frac{B_y}{I}$$

and substituting for K from (25)

$$\eta = \frac{v\lambda\, w\, b}{8\pi^2 ac} \cdot \frac{B_y^2}{EI} \qquad (26)$$

These expressions for K and η approach those of equations (18) and (19) as the leakage factor a tends to unity. In other words the two sets of results are approximately equivalent if it is assumed that the recording field strength drops very little through the thickness of the tape.

MM